Heart Rate Estimation during Intensive Exercise Using Photoplethysmographic Data

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***Abstract*—Wrist photoplethysmograph (PPG) is a noninvasive and continuous approach to acquire the vital sign information such as heartbeats. However, due to the motion artifact from hand movement, the PPG signal obtained from the person who does intensive exercise is always highly contaminated. In this competition, we proposed a framework called SigKnow to calculate the heartbeat rate from the PPG signal. SigKnow first applies several techniques to attenuate the baseline and the noise in the PPG signal. Next, it tries to identify as many peaks as possible for two PPG signals and three accelerometer signals. Then, the pre-processed signals are fed into a peak selection mechanism which will output an estimated heartbeat rate based on several adaptive decision and prediction schemes. The simulations on the 13 datasets with ground truths show that the proposed SigKnow system is very accurate and yields a very less average error of 1.4 beat per minute (BPM).**

***Index Terms*—Photoplethysmograph (PPG); Wearable Computing; Singular Spectrum Analysis; Signal Decomposition; Heart Rate Prediction**

I. INTRODUCTION

HEART rate (HR) monitoring devices are no longer restricted to used in hospitals. They are also used in commercial healthcare product for real time HR estimation. Some HR monitoring devices utilize photoplethysmographic (PPG) signals recorded from wearers' wrists.

Photoplethysmography (PPG) [1, 2] is a non-invasive method to acquire the HR information. It is an optical technique to detect blood volume changes at the skin surface. To generate PPG, a pulse oximeter from the wearable device illuminates the skin with a light-emitting diode. Then the intensity of the reflected light from skin is measured. The variation of the PPG signal is closely related to the cardiac rhythm. Thus, the HR information can be retrieved from the PPG signal. Nevertheless, PPG signals are strongly interfered by the motion artifacts (MA). Therefore, many signal processing methods have been proposed to remove the MA. These methods usually include a signal decomposition process and a peak selection process based on the features of MAs and PPG signals.

López-Silva *et al.* [3] used a heuristic approach which tracks harmonic frequencies of movement to eliminate possible MAs.

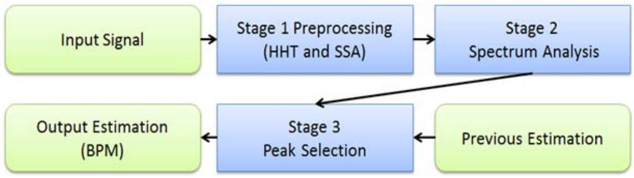

Fig. 1. The three stages of the proposed SigKnow system.

Zhang *et al.* [2] proposed the TROIKA general framework which integrates several stages to complete signal decomposition, sparse signal reconstruction, and spectrum peak tracking and leads to low error rate.

In addition to signal decomposition and peak selection methods, several filters and signal processing techniques have been implemented to reduce the MA problem. For example, a Kalman smoother approach [4] has been proposed to predict the HR. Yan *et al.* [5] developed an MA reduction method utilizing a smoothed pseudo Wigner-Ville distribution.

Most of the methods mentioned above aim to reduce the MA in the non-intensive exercise case, which may not provide high accurate estimation for the HR when intensive exercise happens. The TROIKA framework [2] is one of the approaches focused on intensive fitness.

Motion artifacts are known to contaminate the signals recorded by pulse oximeters, which would make the estimation of the heartbeat rate difficult. With given two pulse oximeters recordings and the measurement of the motion sensor using 3-D accelerometers, our work is to develop an algorithm, which is called SigKnow, to detect actual heart beat rate.

The resulted work, SigKnow, is designed for monitoring the HR in fitness where strong MAs are presented in the wrist PPG signal. There are three main stages in the system: preprocessing, spectrum analysis, and peak selection. The goal of signal decomposition is to eliminate noise without destructing the PPG signal. In spectrum analysis, time-frequency transforms of filtered signals are applied to obtain possible frequency candidates. Then the peak selection mechanism is utilized to estimate the HR by selecting the frequency from the candidate list. The proposed system has a very high accuracy and achieves an error rate near to 1.4 BPM.

II. Algorithm and Framework

There are three main stages in the proposed SigKnow system: (i) preprocessing, (ii) spectrum analysis, and (iii) peak selection, as in Fig. 1.

The purpose of preprocessing is to eliminate as much noise as possible without destroying the original signal. It first utilizes the Hilbert-Huang Transform (HHT) to handle the nonlinear and non-stationary MA in the input PPG signal. Then the signal is further decomposed by singular spectrum analysis (SSA) and independent component analysis (ICA).

In the second stage, spectrum analysis, the time series information is transformed into the frequency domain by the fast Fourier transform (FFT), the windowed FFT, or the periodogram.

In the last stage, we designed an adaptive peak selection mechanism to estimate the HR from (i) the spectrum of the PPG signal, (ii) the spectrum of the accelerometer signal, (iii) the variation of the energy of the accelerometer signal, which helps us to estimate whether the HR will increase or decrease, and (iv) the HR in the previous frame.

A. Preprocessing

1) Hilbert-Huang Transform (HHT)

Unlike the traditional method that uses a band-pass filter for denoising, SigKnow adopts the HHT to pre-filter the PPG signal. The HHT has the advantage of preserving the time domain information in nonlinear and non-stationary signals. It is an adaptive method to analyze the characteristics of a signal in the time-frequency domain and yield a meaningful interpretation [6, 7].

There are two main parts in the HHT [7]: empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). In EMD, signals are separated into several intrinsic mode functions (IMFs). This decomposition method is based on the local time-scale characteristic of the data. Moreover, unlike sinusoid functions, the IMF has neither a fixed amplitude nor a fixed frequency. Therefore, it is applicable to nonlinear and non-stationary data.

Traditional signal analysis methods, such as the FFT, decompose a signal into sinusoidal components. However, many biomedical signals, such as ECG and PPG, are not sinusoidal waves. Directly applying the FFT on the PPG signals and truncating the high- and low-frequency information results in losing some essential information. This problem is similar to the Gibbs phenomenon. To better reconstruct the PPG signal, SigKnow replaced the band-pass filter with the HHT.

After EMD, the original PPG signal is separated into 8 components. Among the 8 components, the first two and the last two components, which correspond to the low frequency and the high frequency parts, are truncated. Then, the PPG signal is reconstructed by adding up the remained 4 components. Fig. 2 shows the frequency spectrums of the reconstructed signal after performing the HHT and the band-pass filter. As shown in Fig. 2(a), HHT method keeps some low-frequency and high-frequency information of the original signal.

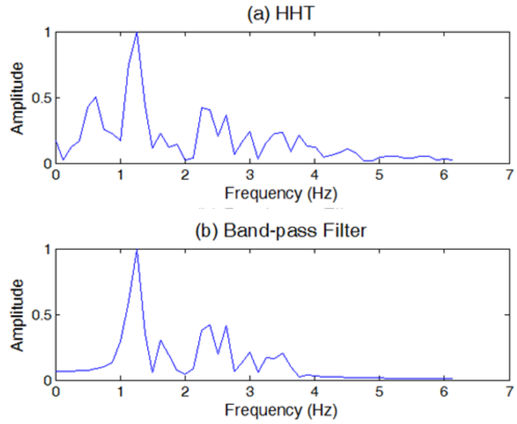

Fig. 2. Signal filtered with (a) the HHT and (b) the band-pass filter.

2) Singular Spectrum Analysis (SSA)

There are 4 steps in total for Singular Spectrum Analysis (SSA) [8]: (i) embedding, (ii) singular value decomposition (SVD), (iii) grouping, and (iv) diagonal averaging.

Embedding is the step that maps a one-dimensional time series $\mathrm{Y} = y_1, y_2, \dots, y_N$ into a two-dimensional matrix X:

$$\mathrm{X} = [x_1, x_2, \dots, x_K] \quad \text{where } x_i = (y_i, \dots, y_{i+L-1})^T, \tag{1}$$

the matrix X is called a Hankel matrix.

Next, we perform SVD on the matrix X. It factorizes a matrix into U, Σ, and V ($\mathrm{X} = \mathrm{U\Sigma V}^*$ where $*$ means conjugate and transpose.) For a given $m \times n$ matrix X, after performing SVD, U is a $m \times m$ unitary matrix, Σ is a $\mathrm{m} \times \mathrm{n}$ diagonal matrix with eigenvalue on the diagonal, and V is an $n \times n$ unitary matrix. Then, X can be decomposed into d components:

$$\mathrm{X} = \mathrm{X}_1 + \mathrm{X}_2 + \cdots + \mathrm{X}_d \tag{2}$$

where d is the rank of X, $\mathrm{X}_i = \sqrt{\lambda_i} U_i V_i^T$, and λ_i is the i^{th} largest eigenvalue. Each set of $(\lambda_i, \mathrm{U}_i, \mathrm{V}_i)$ is called the i^{th} eigentriple.

The main difference between the SSA adopted in the proposed SigKnow system and the traditional SSA process is in the third step, grouping. In this step, k eigenvectors are selected where $1 \le k \le d$. These components are selected based on their similarity with the three-channel accelerometer data.

For the accelerometer data, the windowed periodogram is used to calculate the spectrum of the three accelerometer data. The frequency component with the maximum amplitude A_{max} is called the dominant frequency. If the amplitude A_i of the component f_i is larger than $\mathrm{A}_{max}/2$, it is said to be one of the major components of the acceleration data. These major components are put into a group called $\mathrm{M}f_{acc}$.

Among the d components generated by SVD from the PPG signal, those with the frequency close to any component in $\mathrm{M}f_{acc}$ (within ± 0.2 Hz) are eliminated. However, the frequency of the MA may be close to the fundamental and harmonic frequencies of heartbeats. Thus, the component whose frequency is close to the HR in the previous window will not be deleted. Suppose that $\mathrm{X}_{i(j)}$ (j = 1, 2, …, k) are the undeleted components. Summing them, we obtain the matrix $\mathrm{X_I} = \mathrm{X}_{i(1)} + \mathrm{X}_{i(2)} + \cdots + \mathrm{X}_{i(k)}$.

Diagonal averaging is the last step of SSA. It transforms each Hankel matrix I into a time series z = [z_1, z_2, …, z_N] where z_i equals to the average of $X_I[p, q]$ where p and q satisfy

$$p + q = i + 2. \quad (3)$$

3) Independent Component Analysis (ICA)

Other than SSA, we also tried the method of independent component analysis (ICA) for signal decomposition [9]. ICA is a statistical and computational technique to reveal the hidden factors within a random variable or a deterministic signal.

ICA is to construct a generative model for the observed multivariate data. In the model, the signal is assumed to be a linear mixture of some unknown latent components and the mixing system is also unknown. The latent variables are assumed non-Gaussian and mutually independent and they are called the independent components of the observed data.

The mathematical definition of ICA is as follows. Suppose that there are n signal sources $s_1, s_2, \dots, s_n$, and we observe n linear mixtures $x_1, x_2, \dots, x_n$ of these n signal sources:

$$x_j = a_{j,1}s_1 + a_{j,2}s_2 + \cdots + a_{j,n}s_n \;, j = 1, \dots, n. \quad (4)$$

The above equation can be re-expressed as a matrix form. If the column vectors of **x** are $x_1, x_2, \dots, x_n$, the column vectors of **s** are $s_1, s_2, \dots, s_n$, and the entries of **A** are a_{ij}, then the mixing model can then be written as

$$\mathbf{x} = \mathbf{As}. \quad (5)$$

Then, after estimating the matrix A, we can compute its inverse $\mathbf{W} = \mathbf{A}^{-1}$. Then, the independent component can be simply obtained by:

$$\mathbf{s} = \mathbf{Wx}. \quad (6)$$

ICA is applied to separate noise from the two PPG signals. This idea is based on the assumption that the noise is independent of the PPG signal. SSA has the advantage of better defining the relationship between each PPG signal component and MA frequencies. After applying ICA to convert two PPG signals into two independent vectors, one of the vectors will contain the components which are highly related to the noise. This fact can be considered during peak selection.

B. Spectrum Analysis

Spectrum analysis transforms the preprocessed data into the frequency domain and obtains several frequency candidates where the estimated BPM will be selected from. To avoid the ringing artifact from the sinc function, we replace the rectangular window in the fast Fourier transform (FFT) with an asymmetric Gaussian window:

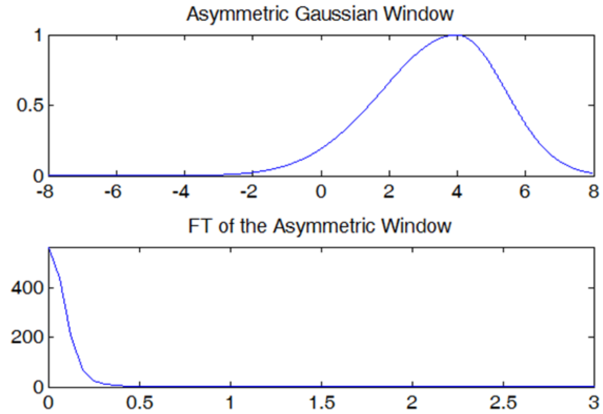

Fig. 3. Asymmetric Gaussian window in (a) the time domain and (b) the frequency domain.

$$y(t) = \begin{cases} \exp\left(-15\,\frac{125(t-12)+0.5}{1500}\right), t \le 12 \\ \exp\left(-4\,\frac{125(t-12)-0.5}{500}\right), t > 12 \end{cases} \quad (7)$$

From Fig. 3, it is clear that there is only a peak near the origin. Compared to the rectangular window, there is only a peak produced by the signal after convolution of two frequency-spectrums. That is, it will reduce the pseudo-peaks from the original rectangular window whose FFT is a sinc function. Therefore, it decreases the number of local maximums (i.e., the frequency candidates) and reduces the computation time of peak selection.

In the spectrum analysis, one may choose all the peaks of the spectrum as the frequency candidates used in the next stage. However, it is inevitable that, if the frequency of the heartbeats is very close to that of the MA, the two components will overlap in the frequency domain and one of the components is missed when selecting frequency peaks. To recover the missing peak, we try to analyze the spectrum and find all the points with different slopes in the left side and the right side. After considering the slope variation, not only the local maxima but also the others points will be considered as the candidates. It is helpful to reduce the error caused from the MA and noise. Moreover, to increase the accuracy of frequency estimation, we adopt the 15,000-point FFT, since the frequency interval is *Fs*/*N* where *Fs* = 125Hz is the sampling frequency and *N* is the number of the points of the FFT.

C. Peak Selection

The proposed peak selection mechanism considers the following features of the signals: the amplitude of each frequency candidate obtained from the PPG signal, the frequency spectrums of the PPG and accelerometer signals, the energy variation of the accelerometer signal, and the HR of the previous window.

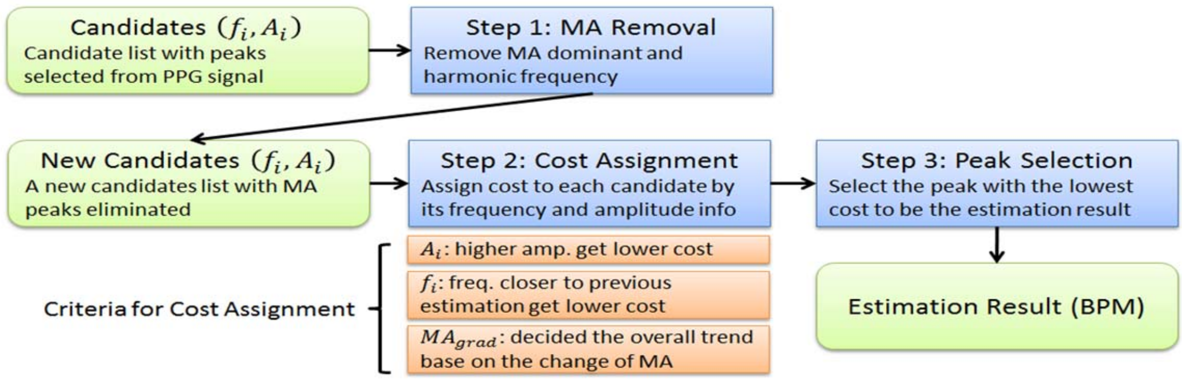

Fig. 4. Flowchart for the proposed peak selection mechanism.

After preprocessing and spectrum analysis, we obtain a group of frequency candidates. Each of them is represented by a tuple (f_i, A_i) where f_i is the frequency and A_i is the amplitude. A cost value is also assigned for every candidate. The peak selection mechanism adjusts the cost of each candidate considering the features mentioned above. The frequency candidate with the least cost value is chosen as the estimated heartbeat rate.

Before assigning a cost value for each candidate, the peak selection mechanism lowers the amplitude A_i of the candidates who have the similar frequency as that of MA, including the dominant and harmonic frequencies. Moreover, if the amplitude A_i is lower than a threshold, then the frequency candidate is removed from the list.

Then, the cost value for each candidate is calculated according to both the frequency value and the amplitude with a linear model:

$$\mathrm{cost}_{total}(i) = c(\mathrm{cost}_{amp}(i) + \sum_{i=1}^{n} \mathrm{cost}_{amp}(i)) + \mathrm{cost}_{freq}(i) \quad (8)$$

where c is some constant, cost_{amp} and cost_{freq} are the costs related to amplitude and frequency, respectively. Both of them are decided by linear models with coefficient λ, α, and β. We obtain cost_{amp} by dividing some constant λ with the amplitude:

$$\mathrm{cost}_{amp}(i) = \frac{\lambda}{A_i}. \quad (9)$$

The larger the amplitude is, the lower the cost is. Therefore, the frequency candidate with higher amplitude has a higher possibility to be selected as the estimated HR. A rather complicated process is designed for calculating cost_{freq}using both the frequency of the candidate, f_i, and the BMP of the previous window $f_{previous}$ or the average BPMs f_{avg} of the previous four windows:

$$\mathrm{cost}_{freq}(i) = \alpha \cdot g(f_i - f_{previous}) + \beta \cdot g(f_i - f_{avg}) \quad (10)$$

where α and β are coefficients and $g(df)$ ($df = f_i - f_{previous}$ or $df = f_i - f_{avg}$) is a linear model shown as in Fig. 5. If the input frequency is far away from the BMPs in previous windows, cost_{freq} will have a higher value. In addition to the previous BMPs, the proposed algorithm also applies the gradient of MA, which is denoted by MA_{grad}, to estimate the HR of the current window. MA_{grad} implies the variation of motion intensity: a higher absolute value means more intense movement. Three models are established for different MA_{grad} which will affect the value of the coefficients α and β. MA_{grad} can be determined from the variation of the energy of the accelerator signal.

Last but not least, since there is no enough previous result for the first four windows, previous estimation and MA_{grad} is not applied for those windows.

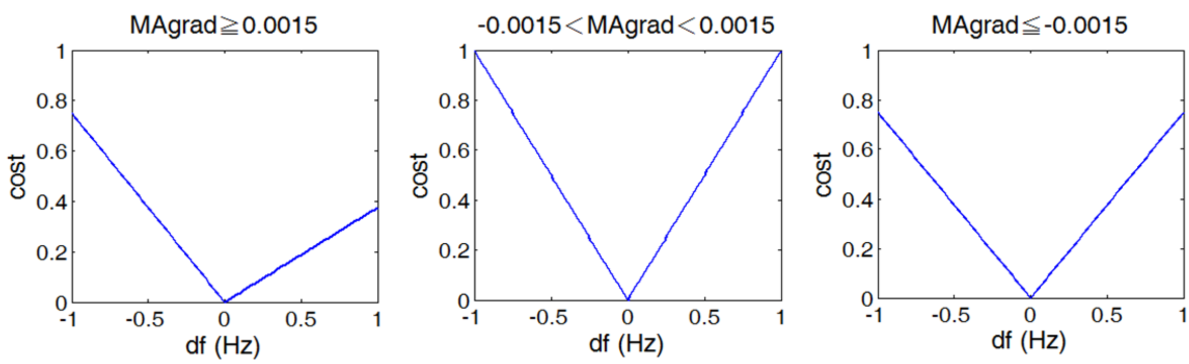

Fig. 5. Linear models used for determining cost_{freq} where df is the difference between f_i and the BMP of the previous window $f_{previous}$ or the average BMPs of the previous four windows f_{avg}. If the motion is getting more and more intense ($\mathrm{MA}_{grad} \geq 0.0015$), one can conclude that the HR should have an increasing trend. Therefore, if the frequency is larger than the HR of the previous window, a smaller cost will be assigned. As for the moderate motion case ($|\mathrm{MA}_{grad}| \leq 0.0015$), the model has a steeper slope since the signal is expected to be less affected by the MA.

III. Experimental Result

A. Data Recording

The training sets we used for SigKnow was provided by Z. Zhang *et al.* [2]. Each dataset corresponds to a subject who performed a variety of physical exercises such as fast running, weightlifting, or jumping and contains a two-channel PPG signal and a three-axis accelerometer signal from his wrist and a one-channel ECG signal from his chest. The ECG signal provides the ground-truth of heart rates. All these signals were recorded simultaneously at the sampling rate of 125 Hz.

Among the 13 datasets, 12 of them were measured while the subjects were running with changing speed: resting for 0.5 minute, running with the speed of 6-8 km/hour for 1 minute, running with 12-15 km/hour for 1 minute, running with 6-8km/hour for 1 minute, running with 12-15 km/hour for 1 minute, and taking a rest for 0.5 minute. The 13^{th} dataset was recorded while the subject was doing push-up.

B. Source Code and Parameter Settings

In our program we revised the peak detection package by Eli Billauer and the Singular Spectrum Analysis smoother by Francisco Javier Alonso. The peak tolerance in peak detection is set to be 0.05. As for the linear model used in peak selection, we found empirically that setting the parameters $(c, \lambda, \alpha, \beta)$ to $(1.5, 10, 8, 10)$ yields the best result.

C. Performance Measurement

Since the performance of an algorithm judged by this competition is the Average Absolute Error (AAE), we will also use it to estimate the in-sample (training sets) performance of the proposed method. The AAE is defined as follows:

$$\text{AAE(a given algorithm)} = \frac{1}{N}\sum_{i=1}^{N} |BPM_{est}(i) - BPM_{true}(i)| \tag{11}$$

where N is the number of time windows, $\text{BPM}_{est}(i)$ is the estimated BPM in the i^{th} time window, and $\text{BPM}_{true}(i)$ is the corresponding ground truth. What we are trying to do is to minimize the AAE of training sets but simultaneously taking robustness into consideration, i.e., keep our model from the over-fitting problem (the problem means performing perfectly on training sets but poorly on testing sets).

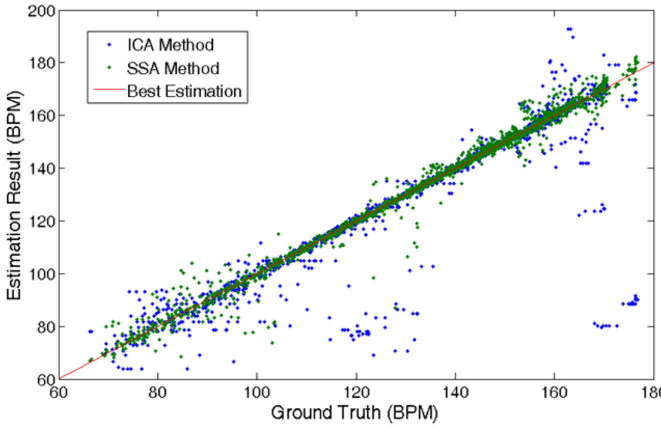

Fig. 6. Estimation result of the ICA method and the SSA method.

D. Result and Discussion

Table 1 shows the BPM estimation error of using different combinations of techniques. According to the average error, decomposing the signal with the HHT and the SSA and using a rectangular window for spectrum analysis yields the best result.

In our study, we found that the HHT acts as a better filter for non-sinusoidal signal. However, since the recursive process is required, it costs more computation time. Therefore, the final version of SigKnow only applies the HHT at the first 10 seconds to correctly initialize the peak selection mechanism.

Another fact we found is that, due to the similarity between the real PPG signal and the MA, SSA is a better method for signal decomposition. As shown in Fig. 6, the ICA method has a higher possibility of over or under estimate the HR. The average error for ICA is 2.9 BPM and that of SSA is 1.4 BPM.

Figs. 7 and 8 shows the overall estimation result of the 5^{th} and the 9^{th} dataset. The proposed SigKnow system successfully finds the initial HR for the dataset and accurately estimates the HR during acceleration. Fig. 9 is a typical case for spectrum analysis and peak selection. Various peaks are selected as frequency candidates, marked by red stars. However, the cost of the candidate whose frequency is similar to the major frequency of the MA is increased. Therefore, the mechanism eventually selected the right peak though its amplitude is lower.

Compared to traditional methods, SigKnow uses different methods to eliminate the noise in the first two stages. This is to avoid signal distortion in the PPG channel and the risk of eliminating the true heartbeat frequency that is close to the frequency of the MA. The proposed SigKnow system also adopts an advanced mechanism to find the best estimated BPM

Table 1. Estimation errors of the provided 13 datasets using different combinations of techniques. HHT* stands for using the HHT at the first 20 second.

	Subj 1	Subj 2	Subj 3	Subj 4	Subj 5	Subj 6	Subj 7	Subj 8	Subj 9	Subj 10	Subj 11	Subj 12	Subj13	AVG
Err (BP +SSA+RFFT)	2.93	4.12	3.67	4.24	3.89	3.89	2.37	4.59	4.45	6.31	2.91	3.75	5.42	4.04
Err (HHT +SSA+RFFT)	1.31	1.10	0.74	1.27	1.01	1.01	0.69	1.07	0.87	2.90	1.53	1.52	3.01	1.39
Err (HHT*+SSA+RFFT)	1.34	1.06	0.74	1.16	1.01	1.02	0.69	1.06	0.86	3.20	1.54	1.52	3.23	1.42
Err (HHT*+ICA+RFFT)	2.10	2.11	1.24	1.51	1.18	1.25	0.99	5.51	6.64	7.95	2.08	1.77	3.39	2.90
Err (HHT*+SSA+GFFT)	1.88	2.30	1.29	1.48	1.18	1.35	1.00	5.73	7.20	10.08	2.09	2.34	3.40	3.18

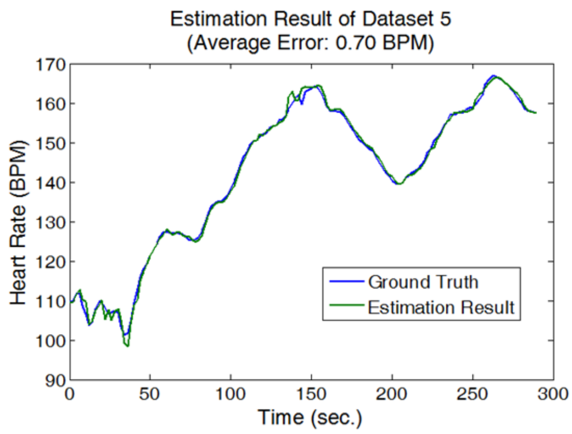

Fig. 7.Overall heartbeat rate estimation result of the 5th dataset.

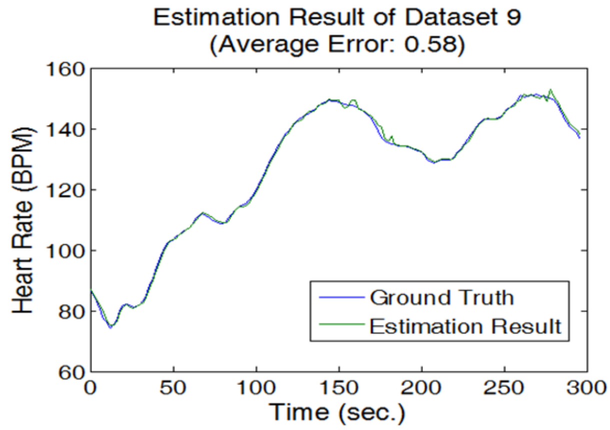

Fig. 8.Overall heartbeat rate estimation result of the 9th dataset.

among all peak candidates. Our delicate peak selection mechanism gives a very satisfactory result and the error is 1.4 BPM in average.

IV. CONCLUSION AND FUTURE WORK

The proposed SigKnow system is robust to estimate the HR from PPG signal during intensive motion. It first preprocesses the input PPG signal and removes the noise using the HHT and SSA. This step is performed carefully to avoid the distortion on the PPG signal. Then the time series is transformed into the frequency domain with the FFT in the second stage, i.e., spectrum analysis. After the FFT, candidates for possible BPM are selected based on the amplitude and the slope in the frequency domain. In the last stage, peak selection, the relative amplitudes of the frequency candidates, the difference between the frequency values and the previous BPM, and the spectrum of the accelerometer signals are used to select the BPM from frequency candidates. Moreover, the MA gradient (i.e., the energy variation of the accelerometer signal) is also applied to estimate the trend of the BPM. For the 13 datasets provided by Z. Zhang *et al.* [2], SigKnow yields a very accurate result with the average error of 1.4 BPM.

Nevertheless, there are still some special cases where the peak selection mechanism may fail to output the right estimation, as in Fig. 10. Moreover, some further improvement can be made for the three stages. During preprocessing, the autocorrelations and cross correlations of PPG signals and accelerometer may be applied to facilitate signal decomposition. In addition to the Fourier transform, other spectrum analysis methods can also be used in the second stage, such as the Gabor transform, the generalized spectrogram, and the wavelet transform. As for the peak selection mechanism, we found that identifying and eliminating the respiratory signal will increase the accuracy of the 10th dataset. However, in some cases, the HR frequency will be misidentified as the respiratory frequency. Better identifying the respiratory peak will increase the accuracy of our peak selection mechanism.

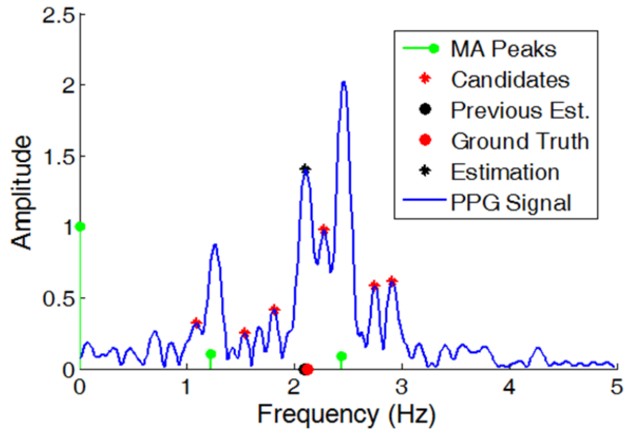

Fig. 9. Frequency spectrum for the 29th window of dataset 5.

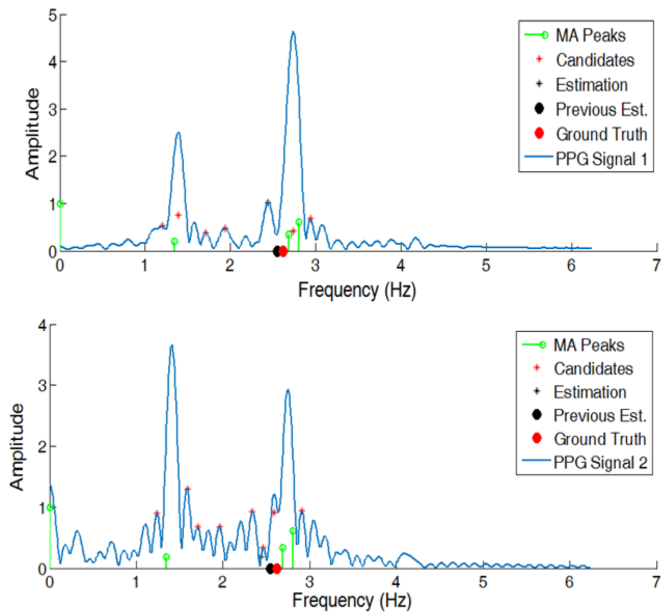

Fig. 10. Special case: The preferred frequency candidate actually has the largest amplitude in PPG signal 1 and the second largest in PPG signal 2. Its location is also close to the previous estimation. However, the preferred candidate has the same frequency as the dominant MA frequency, which makes its cost increased. Therefore, the peak selection mechanism eventually selects the peak near to the previous estimation though its amplitude is far smaller than the preferred one.

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